CHAPTER 4: PROPORTIONS AND EXPRESSIONS

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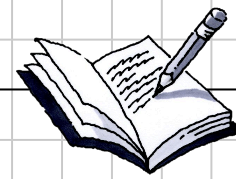

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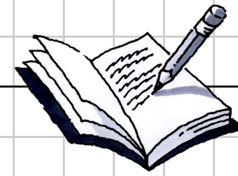

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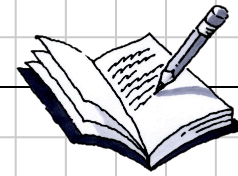

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MATH NOTES

MATHEMATICAL PROPERTIES

When two numbers or variables are combined using addition, the order in which they are added does not matter. For example, $7+5=5+7$. This fact is known as the **Commutative Property of Addition**.

Likewise, when two numbers are multiplied together, the order in which they are multiplied does not matter. For example, $5 \cdot 10 = 10 \cdot 5$. This fact is known as the **Commutative Property of Multiplication**.

These results can be generalized using variables:

$a+b=b+a$ and $a \cdot b = b \cdot a$

Note that subtraction and division do not satisfy the Commutative Property, since $7-5 \neq 5-7$ and $10 \div 5 \neq 5 \div 10$.

When three numbers are added, you usually add the first two of them and then add the third one to that result. However, you could also add the last two together and then add the first one to that result. The **Associative Property of Addition** tells you that the order in which the numbers are added together does not matter. The answer to the problem $(7+5)+9$, for example, is the same as $7+(5+9)$.

Likewise, when three numbers are multiplied together, which pair of numbers are multiplied together first does not matter. For example, $\left(5 \cdot (-6)\right) \cdot 10$ is the same as $5 \cdot (-6 \cdot 10)$. This is the **Associative Property of Multiplication**.

These results can be generalized using variables:

$(a+b)+c=a+(b+c)$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Note that subtraction and division are *not* associative, since:

To multiply $8(24)$, written as $8(20+4)$, you can use the generic rectangle model at right.

The product is found by $8(20)+8(4)$. So $8(20+4)=8(20)+8(4)$. This example illustrates the **Distributive Property**.

Symbolically, for any numbers a, b, and c: $a(b+c)=a(b)+a(c)$.

SIMILARITY

Two figures are **similar** if they have the same shape but not necessarily the same size. In similar figures, the lengths of all corresponding pairs of sides have the same ratio and the measures of corresponding angles are equal.

SCALE FACTOR

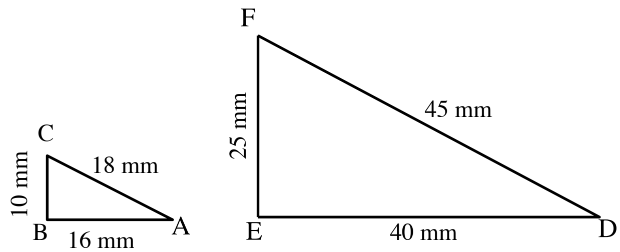
A **scale factor** compares the sizes of the parts of the scale drawing of an object with the actual sizes of the corresponding parts of the object itself. The scale factor in similar figures is the simplified ratio of any pair of corresponding sides.

Example:

$\frac{AB}{DE} = \frac{16}{40} = \frac{2}{5}$

$\frac{BC}{EF} = \frac{10}{25} = \frac{2}{5}$

$\frac{CA}{FD} = \frac{18}{45} = \frac{2}{5}$



The simplified ratio of every pair of corresponding sides is the same. The scale factor is $\frac{2}{5}$.

PROPORTIONAL RELATIONSHIPS

A relationship is **proportional** if one quantity is a multiple of the other. This relationship can be identified in tables, graphs, and equations.

Table: Equivalent ratios of $\frac{y}{x}$ (or $\frac{x}{y}$) can be seen in a table.

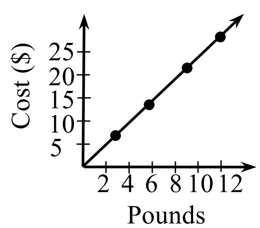
Graph: A straight line through the origin.

Equation: An equation of the form $y = kx$ where k is the **constant of proportionality**.

Example: Three pounds of chicken cost $7.00. What is the cost for x pounds?

Equation: $y = \frac{7}{3}x$

Pounds (x)	0	3	6	9	12
Cost (y)	0	7	14	21	28

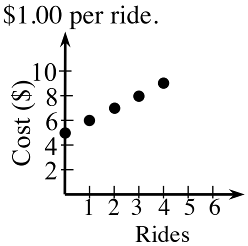


The relationship between pounds and cost is proportional. The table has equivalent ratios ($\frac{7}{3} = \frac{14}{6} = \frac{21}{9}$), the graph is a straight line through the origin, and the equation is of the form $y = kx$.

Example: The county fair costs $5.00 to enter and $1.00 per ride.

Equation: $y = 1x + 5$

Rides (x)	0	1	2	3	4
Cost (y)	5	6	7	8	9



The relationship between rides and cost is not proportional, because the table does not contain equivalent ratios ($\frac{6}{1} \neq \frac{7}{2} \neq \frac{8}{3}$), the graph does not pass through the origin, and the equation contains addition.

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UNIT RATE

A **rate** is a ratio that compares, by division, the amount one quantity changes as another quantity changes.

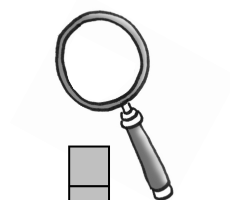
$$\text{rate} = \frac{\text{change in one quantity}}{\text{change in another quantity}}$$

A **unit rate** is a rate that compares the change in one quantity to a one unit change in another quantity. For example, *miles per hour* is a unit rate, because it compares the change in miles to a change of one hour. If an airplane flies 3000 miles in 5 hours and uses 6000 gallons of fuel, you can compute several unit rates.

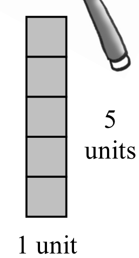
It uses $\frac{6000 \text{ gallons}}{5 \text{ hours}} = \frac{1200 \text{ gallons}}{1 \text{ hour}}$ or $\frac{6000 \text{ gallons}}{3000 \text{ miles}} = \frac{2 \text{ gallons}}{1 \text{ mile}}$.

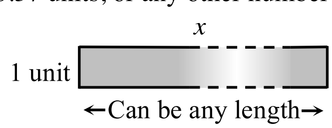
It travels at $\frac{3000 \text{ miles}}{5 \text{ hours}} = \frac{600 \text{ miles}}{1 \text{ hour}}$.

NAMING ALGEBRA TILES

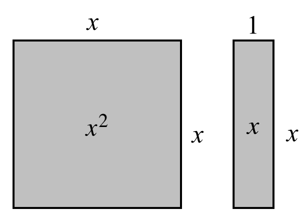


Algebra tiles help us represent unknown quantities in a concrete way. For example, in contrast to a 1×5 tile that has a length of 5 units, like the one shown at right, an x-tile has an unknown length. You can represent its length with a symbol or letter (like x) that represents a number, called a variable. Because its length is not fixed, the x-tile could be 6 units, 5 units, 0.37 units, or any other number of units long.

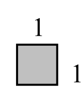




Algebra tiles can be used to build algebraic expressions. The three main algebra tiles are shown at right. The large square has a side of length x units. Its area is x^2 square units, so it is referred to as an x^2-tile.



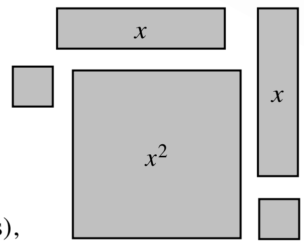
The rectangle has length of x units and width of 1 unit. Its area is x square units, so it is called an x-tile.



The small square has a side of length 1 unit. Its area is 1 square unit, so it is called a one or unit tile. Note that the unit tile in this course will not be labeled with its area.

COMBINING LIKE TERMS

This course uses tiles to represent variables and single numbers (called **constant terms**). Combining tiles that have the same area to write a simpler expression is called **combining like terms**. See the example shown at right.



$x^2 + 2x + 2$

More formally, **like terms** are two or more terms that have the same variable(s), with the corresponding variable(s) raised to the same power.

Examples of like terms: $2x^2$ and $-5x^2$, $4ab$ and $3ab$.

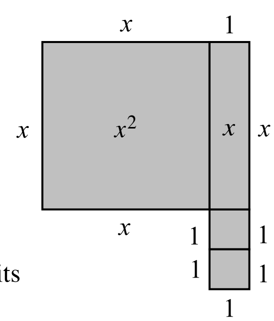
Examples that are *not* like terms: 5 and $3x$, $5x$ and $7x^2$, a^2b and ab.

When you are not working with the actual tiles, it helps to visualize them in your mind. You can use the mental images to combine terms that are the same. Here are two examples:

Example 1: $2x^2 + x + 3 + x^2 + 5x + 2$ is equivalent to $3x^2 + 6x + 5$

Example 2: $3x^2 + 2x + 7 - 2x^2 - x + 7$ is equivalent to $x^2 + x + 14$

When several tiles are put together to form a more complicated figure, the area of the new figure is the sum of the areas of the individual pieces, and the perimeter is the sum of the lengths around the outside. Area and perimeter expressions can be **simplified**, or rewritten, by combining like terms.



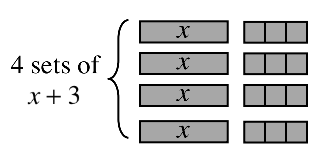
For the figure at right, the perimeter is:
$x + 1 + x + 1 + 1 + 1 + 1 + 1 + x + x = 4x + 6$ units

DISTRIBUTIVE PROPERTY

The **Distributive Property** states that multiplication can be "distributed" as a multiplier of each term in a sum or difference. It is a method to separate or group quantities in multiplication problems. For example, $3(2+4) = 3 \cdot 2 + 3 \cdot 4$. Symbolically, it is written:

$a(b+c) = ab + ac$ and $a(b-c) = ab - ac$

For example, the collection of tiles at right can be represented as 4 sets of $x+3$, written as $4(x+3)$. It can also be represented by 4 x-tiles and 12 unit tiles, written as $4x+12$.



$4(x+3) = 4x + 12$

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